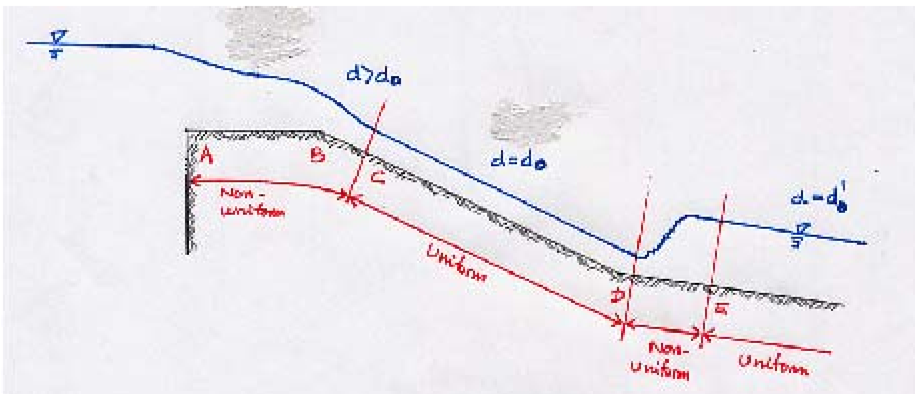
CAPTER Four

UNIFORM FLOW

Concept of Uniform Flow

- Uniform flow is referring the steady uniform flow
- Steady flow is characterized by no changes in time.
- Uniform flow is characterized by the water cross section and depth remaining constant over a certain ***reach*** of the channel.
- For any channel of given roughness, cross section and slope, there exists one and only one water depth, called the ***normal depth*** , at which the flow will be uniform.
- Uniform equilibrium flow can occur only in a straight channel with a constant channel slope and cross-sectional shape, and a constant discharge.
- The energy grade line S_f, water surface slope S_w and channel bed slope S_0 are all parallel, i.e. $S_f=S_w=S_o$

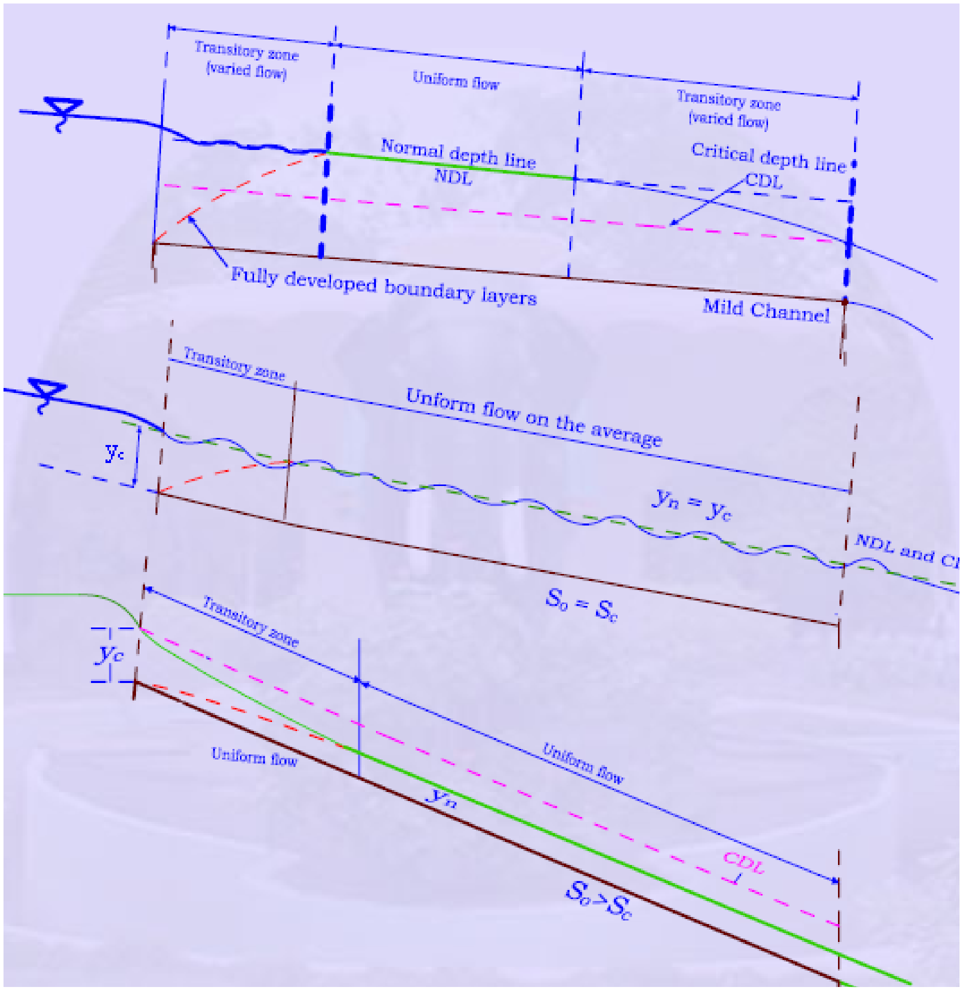


- For steady uniform channel flow, channel slope, depth and velocity all remain constant along the channel

Establishment of Uniform Flow

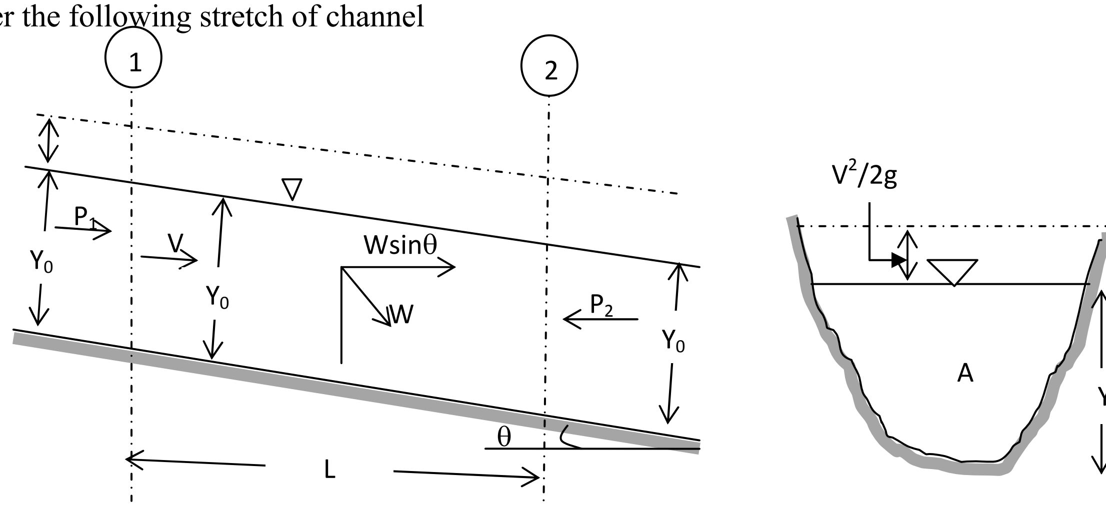
- When flow occurs in an open channel, resistance is encountered by the water as it flows downstream
- A uniform flow will be developed if the resistance is balanced by the gravity forces.
- If the water enters into a channel slowly, the velocity and the resistance are small, and the resistance is outbalanced by the gravity forces, resulting in an accelerating flow in the upstream reach.
- The velocity and the resistance will gradually increase until a balance between resistance and gravity forces is reached. At this moment and afterward the flow becomes uniform.

- The upstream reach that is required for the establishment of uniform flow is known as the ***transitory zone***.



The Chezy Formula

Consider the following stretch of channel



- By definition there is no acceleration in uniform flow
- By applying the momentum equation to control volume encompassing sections 1 and 2, distance L apart as shown in the figure.

$$P_1 - W\sin\theta - F_f - P_2 = M_2 - M_1 \text{ ------------------- (4.1)}$$

Where: P_1 and P_2 are the pressure forces and M_1 and M_2 are the momentum fluxes at section 1 and 2 respectively

W= weight to fluid in the control volume and

F_f = shear force at the boundary

Since the flow is uniform $P_1 = P_2$ and $M_1 = M_2$ also $W = \gamma AL$ and $F_f = \tau_0 PL$

Where τ_0 = average shear stress on the wetted perimeter of length P

γ= unit weight of water

Replacing $\sin\theta$ by S0 (bottom slope) equation (4.1) become

$$\gamma ALS_0 = \tau_0 PL \Rightarrow \tau_0 = \gamma\frac{A}{P}S_0 = \gamma RS_0 \text{ ------------------ (4.2)}$$

Where R= A/P = hydraulic radius , which is a length parameter accounting for the shape of the channel. And it plays a very important role in developing flow equations which are common to all shapes of channels.

Expressing the average shear stress $\tau_0 = k\rho V2$, where k=a coefficient which depends on the nature of the surface and flow parameters. Equation (4.2) can be written as

$$k\rho V^2 = \gamma RS_0 \Rightarrow V = C\sqrt{RS_0} \text{ ------------------------------ (4.3)}$$

Where $C = \sqrt{\frac{\gamma}{\rho}\frac{1}{k}}$ = a coefficient which depends on the nature of the surface

Equation (4.3) is known as ***Chezy formula*** and the coefficient C is known as the ***chezy coefficient.***

We remember that for pipe flow, the Darcy –Weisbach equation is

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Where h_f = head loss due to friction in a pipe of diameter D and length L

f = Darcy-Weisbach friction factor

For smooth pipes, f is found to be a function of the Reynolds number ($\mathrm{Re} = \frac{VD}{\nu}$) only. For rough turbulent flows, f is a function of the relative roughness (ε/D) and types of roughness, which independent of the Reynolds number.

So for the case of Open Channel, we can be considered to be a conduit cut into two. The hydraulics radius would then be appropriate length parameter and prediction of friction factor f. So equation of head loss due to friction is written as

$$h_f = f\frac{L}{4R}\frac{V^2}{2g} \Rightarrow V = \sqrt{\frac{8g}{f}}.\sqrt{R}.\sqrt{h_f / L} \quad \text{-------------------- (4.4)}$$

Note that for uniform flow in an open channel h_f/L = slope of the energy line = $S_f = S_0$, it may be seen that equation (4.4) is the same as Chezy formula, equation (4.3). with

$$C = \sqrt{8g / f} \quad \text{--(4.5)}$$

Equation (4.5) be use to develop different charts and empirical formulas that relate C with Re.

$$\left(\mathrm{Re} = \frac{4RV}{\nu}\right) and \left(\frac{4R}{\varepsilon_s}\right)$$

$$\frac{1}{\sqrt{f}} = 1.80\log\mathrm{Re} - 1.5146 \quad \text{--- (4.6)}$$

And $$\frac{1}{\sqrt{f}} = 1.14 - 2.0\log\left(\frac{\varepsilon_s}{4R} + \frac{21.25}{\mathrm{Re}^{0.9}}\right) \quad \text{-- (4.7)}$$

Equation (4.7) is valid for $5000 \leq \mathrm{Re} \leq 10^8$ and $10^{-6} < \varepsilon_s/4R < 10^{-2}$

Generally, the open channels that are encountered in the field are very large in size and also in the magnitude of roughness elements. Due to scarcity of reliable experimental or field data on channels covering a wide range of parameters, values of ε_s are not available to the same degree of confidence as for pipe materials. However, the following table will give the estimated values of ε_s for some common open channel surfaces.

Table 4. values of ε_s for some common open channel surfaces

S.No	Surface	Equivalent Roughness (ε_s) in mm
1	Glass	$3x10^{-4}$
2	Very smooth concrete surface	0.15 – 0.30
3	Rough concrete	3.0 - 4.5
4	Earth Channels (straight, uniform)	3.0
5	Rubble masonry	6.0
6	Untreated channel	3.0 -10.0

Example 4.1: A 2.0m wide rectangular channel carries water at 20°c at a depth of 0.5m. The channel is laid on a slope of 0.0004. Find the hydrodynamic nature of the surface if the channel is made of

A. Very smooth concrete
B. Rough concrete
C. Estimate the discharges in the channel in both case using chezy formula with Dancy-Weisbach f.

Solution:

B=2.0m

Y=0.5m

$S_0 = 0.0004$

T= 20°c

R= A/P = (0.5*2)/(2+2*0.5)=0.33m

$\iota_o = \gamma R S_o = (9.81x10^3)x0.333x0.0004 = 1.308 N/m^2$

$v_* =$ shear velocity $= \sqrt{\iota_o/\rho} = \sqrt{(1.308/10^3)} = 0.03617$m/sec

Kinematic viscosity (ν) @ 20°c = $10^{-6} m^2/s$

a). For a smooth concrete surface

from the Equivalent roughness table, $\varepsilon_s = 0.25mm = 0.00025m$

$\varepsilon_s v_*/\nu = (0.00025*0.03617)/(10^{-6}) = 9.04 > 4$ but less than 60 $\Rightarrow$ Transition

b) for a rough concrete

From the Equivalent roughness table, $\varepsilon_s = 3.5mm = 0.0035m$

$\varepsilon_s v_*/\nu = (0.0035x0.03617)/(10^{-6}) = 126.6 > 60 \Rightarrow$ Rough

c). case (i) = smooth concrete channel

$\varepsilon_s = 0.25mm$ and $\varepsilon_s/4R = (0.25)/(4x0.33x10^3) = 1.894x\ 10^{-4}$

$$\frac{1}{\sqrt{f}} = 1.14 - 2.0\log\left(\frac{\varepsilon_s}{4R} + \frac{21.25}{\mathrm{Re}^{0.9}}\right)$$

Since Re is known we use the moody chart or iterative trial and error methods, we get f= 0.0145

C= √(8g/f) = √(8x9.81/0.0145) = 73.6

V = C√(RS_0) = 73.6x√(0.333x0.0004) = 0.850m/s

Q= AV= (2x0.5)x0.850 = 0.85m^3/se

Case (ii) Rough concrete Channel

ε_s= 3.5mm and ε_s/4R = (3.5)/(4x0.33x10^3) = 2.625x 10^{-3}

f = 0.025

C = √(8g/f) = √(8x9.81/0.025) = 56.0

V = C√(RS_0) = 56x√(0.333x0.0004) = 0.647m/s

Q= AV= (2x0.5)x0.647 = 0.647m^3/se

The MANNING'S Formula

The simplest resistance formula and the most widely used equation for the mean velocity calculation is the ***Manning equation*** which has been derived by Robert Manning (1890) by analyzing the experimental data obtained from his own experiments and from those of others. His equation is,

$$V = \frac{1}{n} R^{2/3} S_o^{1/2} \quad \text{-----} \quad (4.8)$$

Where V = mean velocity

R= Hydraulic Radius

So = channel slope

n = Manning's roughness coefficient

if we equating Equations. (4.3) and (4.8), we get

$$CR^{1/2} S_o^{1/2} = \frac{1}{n} R^{2/3} S_o^{1/2}$$

$$C = \frac{R^{1/6}}{n} \quad \text{-----} \quad (4.9)$$

Similarly we can be equating equations (4.5) and (4.9) we get

$$\sqrt{\frac{8g}{f}} = \frac{R^{1/6}}{n} \Rightarrow f = \left(\frac{n^2}{R^{1/3}}\right)(8g) \quad \text{-----} \quad (4.10)$$

From equation (4.10) we see $f \alpha \left(\frac{n^2}{R^{1/3}}\right)$, it follows that $n \alpha \varepsilon_s^{1/6}$. If so the both Manning's formula and Darcy – Weisbach formula represent rough turbulent flow at $\frac{\varepsilon v_*}{\nu} > 60$

OTHERS RESISTANCE FORMULAE

Several forms for the chezy coefficient C have been proposed by different investigators in the past. Some of them are in use for problems in open channel flow.

1. Pavlovsik formula: $C = \frac{R^x}{n}$ in which $x = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}\left(\sqrt{n} - 0.10\right)$ and n= manning's coefficient. This formula appears to be in use in Russia
2. Bazin's formula $C = \frac{87.0}{1 + M/R}$, in which M = a coefficient dependent on the surface roughness
3. Ganguillet and Kutter Formula $C = \frac{23 + \frac{1}{n} + \frac{0.00155}{S_o}}{1 + \left[23 + \frac{0.00155}{S_o}\right]\frac{n}{\sqrt{R}}}$, in which n= manning's coefficient

MANNING'S Roughness Coefficient (n)

In applying the Manning equation, the greatest difficulty lies in the determination of the roughness coefficient, n; there is no exact method of selecting the n value. Selecting a value of n actually means to estimate the resistance to flow in a given channel, which is really a matter of intangibles.(Chow, 1959) .To experienced engineers, this means the exercise of engineering judgment and experience; for a new engineer, it can be no more than a guess and different individuals will obtain different results.

Factors Affecting Manning's Roughness Coefficient

It is not uncommon for engineers to think of a channel as having a single value of n for all occasions. Actually, the value of n is highly variable and depends on a number of factors. The factors that exert the greatest influence upon the roughness coefficient in both artificial and natural channels are described below.

a) **Surface Roughness:** The surface roughness is represented by the size and shape of the grains of the material forming the wetted perimeter. This usually considered the only factor in selecting the roughness coefficient, but it is usually just one of the several factors. Generally, fine grains result in a relatively low value of n and coarse grains in a high value of n.

b) **Vegetation:** Vegetation may be regarded as a kind of surface roughness, but it also reduces the capacity of the channel. This effect depends mainly on height, density, and type of vegetation.

c) **Channel Irregularity:** Channel irregularity comprises irregularities in wetted perimeter and variations in cross-section, size, and shape along the channel length.

d) **Channel Alignment:** Smooth curvature with large radius will give a relatively low value of n, whereas sharp curvature with severe meandering will increase n.

e) **Silting and Scouring:** Generally speaking, silting may change a very irregular channel into a comparatively uniform one and decrease n, whereas scouring may do the reverse and increase n.

f) **Obstruction:** The presence of logjams, bridge piers, and the like tends to increase n.

g) **Size and Shape of the Channel:** There is no definite evidence about the size and shape of the channel as an important factor affecting the value of n.

h) **Stage and Discharge:** The n value in most streams decreases with increase in stage and discharge.

i) **Seasonal Change:** Owing to the seasonal growth of aquatic plants, the value of n may change from one season to another season.

Determination of Manning's Roughness Coefficient

a. Cowan Method

Taking into account primary factors affecting the roughness coefficient, Cowan (1956) developed a method for estimating the value of n. The value of n may be computed by,

$$n = (n_0 + n_1 + n_2 + n_3 + n_4)\times m \quad \text{--}(4.11)$$

Where: n_0 is a basic value for straight, uniform, smooth channel in the natural materials involved,

n_1 is a value added to n_0 to correct for the effect of surface irregularities,

n_2 is a value for variations in shape and size of the channel cross-section,

n_3 is a value of obstructions,

n_4 is a value for vegetation and flow conditions, and

m is a correction factor for meandering of channel.

These coefficients are given in Table (4.2) depending on the channel characteristics. (French,1994).

Table 4.2: Values of the manning's coefficients of Cowan Method

Channel conditions			Values
Material involved	Earth	n_0	0.020
	Rock cut		0.025
	Fine gravel		0.024
	Coarse gravel		0.028
Degree of irregularity	Smooth	n_1	0.000
	Minor		0.005
	Moderate		0.010
	Severe		0.020
Variations of channel cross section	Gradual	n_2	0.000
	Alternating occasionally		0.005
	Alternating frequently		0.010–0.015
Relative effect of obstructions	Negligible	n_3	0.000
	Minor		0.010–0.015
	Appreciable		0.020–0.030
	Severe		0.040–0.060
Vegetation	Low	n_4	0.005–0.010
	Medium		0.010–0.025
	High		0.025–0.050
	Very high		0.050–0.100
	Minor		1.000

b. Empirical Formulae for n

Many empirical formulae have been presented for estimating manning's coefficient n in natural streams. These relate n to the bed-particle size. (Subramanya, 1997). The most popular one under this type is the Strickler formula,

$$n = \frac{d_{50}^{1/6}}{21.1} \quad \text{-----------} \quad (4.12)$$

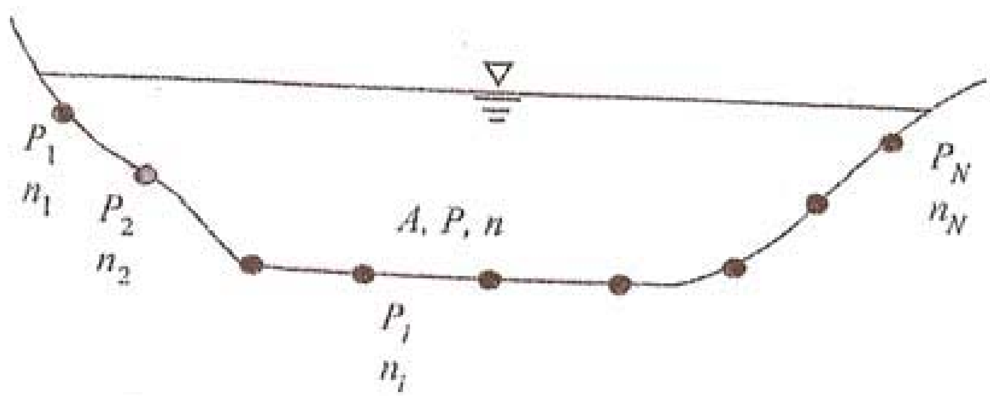
Where d_{50} is in meters and represents the particle size in which 50 per cent of the bed material is finer. For mixtures of bed materials with considerable coarse-grained sizes,

$$n = \frac{d_{90}^{1/6}}{26} \quad \text{-----------} \quad (4.13)$$

Where d90 = size in meters in which 90 per cent of the particles are finer than d_{90}. This equation is reported to be useful in predicting n in mountain streams paved with coarse gravel and cobbles.

c. **Equivalent Roughness**

In some channels different parts of the channel perimeter may have different roughnesses. Canals in which only the sides are lined, laboratory flumes with glass walls and rough beds, rivers with sand bed in deepwater portion and flood plains covered with vegetation, are some typical examples. For such channels it is necessary to determine an equivalent roughness coefficient that can be applied to the entire cross-sectional perimeter in using the Manning's formula. This ***equivalent roughness***, also called the ***composite roughness***, represents a weighted average value for the roughness coefficient, n.



A large number of formulae, proposed by various investigators for calculating equivalent roughness of multi-roughness channel are available in literature. All of them are based on some assumptions and approximately effective to the same degree. We see the derivation of one of the common formula called Horton's Method and present others as table below.

Horton's method of Equivalent Roughness Estimation:

Consider a channel having its perimeter composed of N types rough nesses. P1, P2,…., PN are the lengths of these N parts and n_1, n_2,…….., n_N are the respective roughness coefficients as presented in the above figure.

Let each part Pi be associated with a partial area Ai such that,

$$\sum_{i=1}^{N} A_i = A_1 + A_2 + \ldots\ldots + A_i + \ldots\ldots + A_N = A = \textbf{Total area}$$

It is assumed that the mean velocity in each partial area is the mean velocity V for the entire area of flow,

$$V_1 = V_2 = \ldots\ldots = V_i = \ldots\ldots\ldots = V_N = V$$

By the Manning's equation,

$$S_0^{1/2} = \frac{V_1 n_1}{R_1^{2/3}} = \frac{V_2 n_2}{R_2^{2/3}} = \ldots\ldots = \frac{V_i n_i}{R_i^{2/3}} = \ldots\ldots = \frac{V_N n_N}{R_N^{2/3}} = \frac{Vn}{R^{2/3}} \quad \text{, ---------------------(4.14)}$$

Where n = Equivalent roughness.

From Equ. (4.14),

$$\left(\frac{A_i}{A}\right)^{2/3} = \frac{n_i P_i^{2/3}}{n P^{2/3}}$$

$$A_i = A \frac{n_i^{3/2} P_i}{n^{3/2} P}$$

$$\sum A_i = A = A \frac{\sum \left(n_i^{3/2} P_i\right)}{n^{3/2} P}$$

$$n = \frac{\left(\sum n_i^{3/2} P_i\right)^{2/3}}{P^{2/3}} \quad \text{-- (4.15)}$$

This equation gives a means of estimating the equivalent roughness of a channel having multiple roughness types in its perimeters

Table 4.3. Equations for equivalent Roughness Coefficient (adopt from K. Subramanya 2010)

No	Investigators	n_e	Concept
1	Horton (1933); Einstein (1934)	$= \left[\frac{1}{P}\sum\left(n_i^{3/2} P_i\right)\right]^{2/3}$	Mean Velocity is constant in all subareas
2	Pavloskii (1931), Muhlhofer(1933) Einstein and Banks (1950)	$= \left[\frac{1}{P}\sum\left(n_i^{2} P_i\right)\right]^{1/2}$	Total resistance force F is sum of subarea resistance force, ΣF_i
3	Lotter (1932)	$= \frac{PR^{5/3}}{\sum \frac{P_i R_i^{5/3}}{n_i}}$	Total discharge is sum of subarea discharge
4	Yen (1991)	$= \frac{\sum\left(n_i P_i\right)}{P}$	Total shear velocity is weighted sum of subarea shear velocity

Table 4.4: values of Manning's Roughness coefficient (adopt from ERA design manuals 2001)

Type of Channel and Description	Minimum	Normal	Maximum
EXCAVATED OR DREDGED			
a. Earth, straight and uniform			
1. Clean, recently completed	0.016	0.018	0.020
2. Clean, after weathering	0.018	0.022	0.025
3. Gravel, uniform section, clean	0.022	0.025	0.030
4. With short grass, few weeds	0.022	0.027	0.033
b. Earth, winding and sluggish			
1. No vegetation	0.023	0.025	0.030
2. Grass, some weeds	0.025	0.030	0.033
3. Dense Weeds or aquatic plants in deep channels	0.030	0.035	0.040
4. Earth bottom and rubble sides	0.025	0.030	0.035
5. Stony bottom and weedy sides	0.025	0.035	0.045
6. Cobble bottom and clean sides	0.030	0.040	0.050
c. Backhoe-excavated or dredged			
1. No vegetation	0.025	0.028	0.033
2. Light brush on banks	0.035	0.050	0.060
d. Rock cuts			
1. Smooth and uniform	0.025	0.035	0.040
2. Jagged and irregular	0.035	0.040	0.050
e. Channels not maintained, weeds and brush uncut			
1. Dense weeds, high as flow depth	0.050	0.080	0.120
2 Clean bottom, brush on sides	0.040	0.050	0.080
3. Same, highest stage of flow	0.045	0.070	0.110
4. Dense brush, high stage	0.080	0.100	0.140
NATURAL STREAMS			
1 Minor streams (top width at flood stage < 30 m)			
a. Streams on Plain			
1. Clean, straight, full stage, no rims or deep pools	0.025	0.030	0.033
2. Same as above, but more stones and weeds	0.030	0.035	0.040
3. Clean, winding, some pools and shoals	0.033	0.040	0.045
4. Same as above, but some weeds and stones	0.035	0.045	0.050
5. Same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
6. Same as 4, but more stones	0.045	0.050	0.060
7. Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
8 Very weedy reaches, deep pools, or floodways with heavy stand of timber and underbrush	0.075	0.100	0.150
b. Mountain streams, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stages			
1. Bottom: gravel, cobbles, and few boulders	0.030	0.040	0.050
2. Bottom: cobbles with large boulders	0.040	0.050	0.070
2 Flood Plains			
a. Pasture, no brush			
1. Short grass	0.025	0.030	0.035
2. High grass	0.030	0.035	0.050

Type of Channel and Description	Minimum	Normal	Maximum
b. Cultivated area			
1. No crop	0.020	0.030	0.040
2. Mature row crops	0.025	0.035	0.045
3. Mature field crops	0.030	0.040	0.050
c. Brush			
1. Scattered brush, heavy weeds	0.035	0.050	0.070
2. Light brush and trees in winter	0.035	0.050	0.060
3. Light brush and trees, in summer	0.040	0.060	0.080
4. Medium to dense brush, in winter	0.045	0.070	0.110
5. Medium to dense brush, in summer	0.070	0.100	0.160
d. Trees			
1. Dense willows, summer, straight	0.110	0.150	0.200
2. Cleared land with tree stumps, no sprouts	0.030	0.040	0.050
3. Same as above, but with heavy growth of spouts			
4. Heavy stand of timber, a few down trees, little undergrowth, flood stage below branches	0.050	0.060	0.080
5. Same as above, but with flood stage reaching branches	0.080	0.100	0.120
3 Major Streams (top width at flood stage > 30 m). The n value is less than that for minor streams of similar description, because banks offer less effective resistance.	0.100	0.120	0.160
a. Regular section with no boulders or brush			
b. Irregular and rough section			
4 Various Open Channel Surfaces			
a. Concrete	0.025	--	0.060
b. Gravel bottom with:	0.035	--	0.100
Concrete			
Mortared stone	0.012-	0.020	
Riprap			
c. Natural Stream Channels	0.020		
Clean, straight stream	0.023		
Clean, winding stream	0.033		
Winding with weeds and pools			
With heavy brush and timber	0.030		
d. Flood Plains	0.040		
Pasture	0.050		
Field Crops	0.100		
Light Brush and Weeds			
Dense Brush	0.035		
Dense Trees	0.040		
	0.050		
	0.070		
	0.100		

Example 4.2: An earthen trapezoidal channel (n = 0.025) has a bottom width of 5.0 m, side slopes of 1.5 horizontal: 1 vertical and a uniform flow depth of 1.10 m. In an economic study to remedy excessive seepage from the canal two proposals, a) to line the sides only and, b) to line the bed only are considered. If the lining is of smooth concrete (n = 0.012), calculate the equivalent roughness in the above two cases.

Solution :

Earthen (n) = 0.025

B = 5.0m

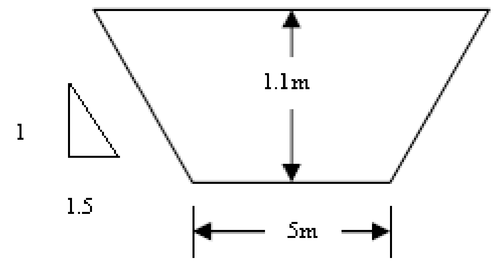
m= 1.5

y = 1.10m

a). line the sides only

b). line the bed only

lining (n) = 0.012



Case a): Lining on the sides only,

For the bed → n1 = 0.025 and P1 = 5.0 m.

For the sides → n2 = 0.012 and $P_2 = 2 \times 1.10 \times \sqrt{1+1.5^2} = 3.97m$

$$P = P_1 + P_2 = 5.0 + 3.97 = 8.97m$$

$$n = \frac{\left(\sum n_i^{3/2} P_i\right)^{2/3}}{P^{2/3}} \qquad n = \frac{\left[5.0 \times 0.025^{1.5} + 3.97 \times 0.012^{1.5}\right]^{2/3}}{8.97^{2/3}} = 0.020$$

Case b): Lining on the bottom only

$$P_1 = 5.0 \text{ m} \rightarrow n_1 = 0.012$$
$$P_2 = 3.97 \text{ m} \rightarrow n_2 = 0.025 \rightarrow P = 8.97 \text{ m}$$

$$n = \frac{\left(5.0 \times 0.012^{1.5} + 3.97 \times 0.025^{1.5}\right)^{2/3}}{8.97^{2/3}} = 0.018$$

UNIFORM FLOW COMPUTATION

The basic equations involved in the computation of uniform flow are the manning's and the continuity equation. From continuity equation Q = AV, if we substitute the velocity from the manning's formula we get the equation for discharge as

$$Q = \frac{1}{n} A R^{2/3} S_o^{1/2} \text{ -- (4.16)}$$

$$Q = K\sqrt{S_o} \text{ -- (4.17)}$$

Where $K = \frac{1}{n} A R^{2/3}$ is called the conveyance of the channel and express the discharge capacity of the channel per unit longitudinal slope. The term nK = $AR^{2/3}$ is also called the section factor for uniform flow computations.

The basic variables in uniform flow problems can be the discharge Q, velocity of flow V, normal depth y0, roughness coefficient n, channel slope S0 and the geometric elements (e.g. B and side slope m for a trapezoidal channel). There can be many other derived variables accompanied by corresponding relationships. From among the above, the following five types of basic problems are recognized.

Table 4.5 Problem Types and the given and required variables in uniform flow

Problem Type	Given	Required
1	y_0, n, S_0, Geometric elements	Q and V
2	Q, y_0, n, Geometric elements	S_0
3	Q, y_0, S_0, Geometric elements	n
4	Q, n, S_0, Geometric elements	y_0
5	Q, y_0, n, S_0, Geometry	Geometric elements

Hydraulic Radius

Hydraulic radius plays a prominent role in the equations of open-channel flow and therefore, the variation of hydraulic radius with depth and width of the channel becomes an important consideration. This is mainly a problem of section geometry.

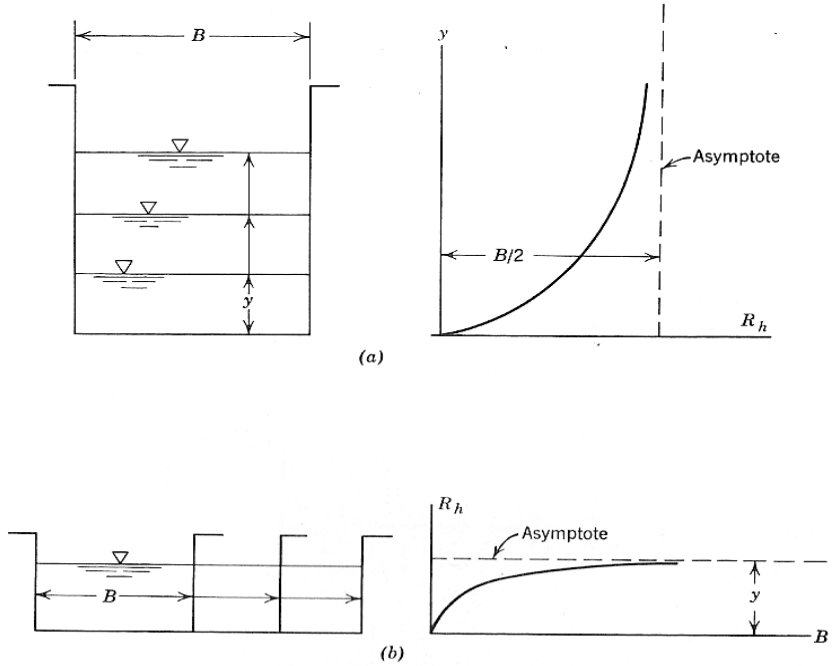


Figure on the relation of Hydraulic Radius with depth and width

Consider first the variation of hydraulic radius with depth in a rectangular channel of width B

$$A = By, P = B + 2y$$

$$R = \frac{A}{P} = \frac{By}{B + 2y}$$

$$R = \frac{B}{\frac{B}{y} + 2}$$

For $$y = 0 \rightarrow R = 0$$
$$y \rightarrow \infty, R = \frac{B}{2}$$

Therefore the variation of R with y is as shown in Fig (a) above. From this comes a useful engineering approximation:

for *narrow deep cross-sections R≈ B/2.* Since any (nonrectangular) section when deep and narrow approaches a rectangle, when a channel is deep and narrow, the hydraulic radius may be taken to be half of mean width for practical applications.

Consider the variation of hydraulic radius with width in a rectangular channel of with a constant water depth y

$$R = \frac{By}{B + 2y}$$

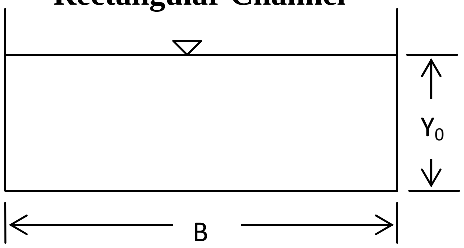
$$R = \frac{y}{1 + \frac{2}{yB}}$$

$$B = 0 \rightarrow R = 0$$
$$B \rightarrow \infty, R \rightarrow y$$

From this it may be concluded that for *wide **shallow rectangular cross-sections R≈y;*** for rectangular sections the approximation is also valid if the section is wide and shallow, here the hydraulic radius approaches the mean depth.

Normal Depth

Rectangular Channel



Area $A = By_o$

Wetted Perimeter $P = B + 2y_o$

Hydraulic Radius $R = A/P$

$$R = \frac{y_o}{1 + 2\frac{y_o}{B}} \text{ ---------- (4.18)}$$

a). wide Rectangular Channel

As yo/B, the aspect ratio of the channel decrease, $R \rightarrow y_o$. Such channels with large bed-widths as compared to their respective depths are known as ***wide rectangular channels***. In these channels, the hydraulics radius approximates to the depth of flow.

Considering a unit width of a wide rectangular channel, $A = y_o$, $R = y_o$ and $B = 1.0$

Discharge per unit width Q/B = $q = \frac{1}{n} y_o^{5/3} S_o^{1/2} \Rightarrow y_o = \left[\frac{qn}{\sqrt{S_o}}\right]^{3/5}$ -------------------------- (4.19)

This approximation of a wide rectangular channel is found applicable to rectangular channels $y_o/B < 0.02$.

b). Rectangular Channels with $yo/B \geq 0.02$

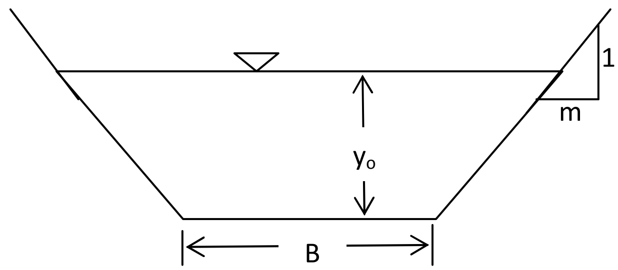
For these channels $\frac{Qn}{\sqrt{S_o}} = AR^{2/3}$, and $AR^{2/3} = \frac{(By_o)^{5/3}}{(B + 2y_o)^{2/3}} = \frac{(y_o/B)^{5/3}}{(1 + 2y_o/B)^{2/3}} B^{8/3}$

$$\frac{Qn}{\sqrt{S_o}B^{8/3}} = \frac{AR^{2/3}}{B^{8/3}} = \frac{(\eta_o)^{5/3}}{(1 + 2\eta_o)^{2/3}} = \phi(\eta_o) \text{ -- (4.20)}$$

Where $\eta_0 = \frac{y_0}{B}$, Tables of $\phi(\eta o)$ Vs ηo will provide a non-dimensional graphical aid for general application. Since $\phi = \frac{Qn}{\sqrt{S_o}B^{8/3}}$, one can easily find y_o/B from this table for any combination of Q, n, S_o, and B in a rectangular channel.

Trapezoidal Channel

Following a procedure similar to the above, for a trapezoidal section of side slope m:1.



Area = A = (B + myo)yo
Wetted Perimeter = P= (B +2yo√m2+1)
Hydraulic Radius

$$R = \frac{A}{P} = \frac{(B + my_o)y_o}{\left(B + 2\sqrt{m^2+1}\,y_o\right)}$$

$$\frac{Qn}{\sqrt{S_o}} = AR^{2/3} = \frac{(B + my_o)^{5/3}\, y_o^{5/3}}{\left(B + 2\sqrt{m^2+1}\,y_o\right)^{2/3}}$$, Non-dimensionalising the variables,

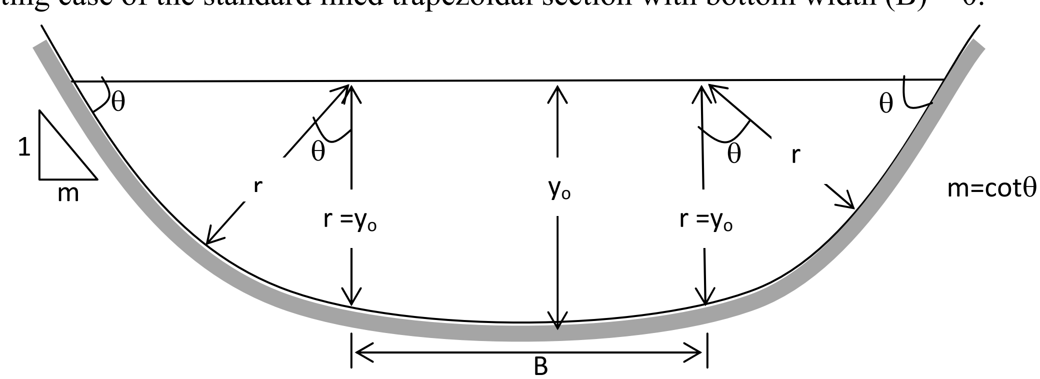
$$\frac{AR^{2/3}}{B^{8/3}} = \frac{Qn}{\sqrt{S_o}\,B^{8/3}} = \frac{(1 + m\eta_o)^{5/3}\,\eta_o^{5/3}}{\left(1 + 2\sqrt{m^2+1}\,y_o\right)^{2/3}} = \phi(\eta_o, m) \quad (4.21)$$

Where $\eta_o = y_o/B$.

Equation (4.21) represent as curves or tables of ϕ vs ηo with m as the third parameter to provide a general normal depth solution aid. It may be noted m=0 is the case of rectangular.

Lined canal Section

Most lined channels and built-up channels can withstand erosion and reduce seepage. Exposed hard surface lining using materials, such as cement concrete, brick tiles, asphaltic concrete and stone masonry are the members of lined canal category and also carry large canal. Indian Standards (IS: 4745 -1968) consists of two standardized lined canal section (i.e., Trapezoidal and triangular with corners rounded off for discharge > 55m3/sec and < 55m3/sec). Actually the triangular section is the limiting case of the standard lined trapezoidal section with bottom width (B) = 0.



Referring the above figure the full supply depth = normal depth at design discharge = y_o at normal depth

For standard trapezoidal case

Area = A = $By_o + my_o^2 + y_o^2\theta = (B + y_o\varepsilon)y_o$, where $\varepsilon = m + \theta = \left(m + \tan^{-1}\frac{1}{m}\right)$

Wetted perimeter =P = $B + 2my_o + 2y_o\theta = B + 2y_o\varepsilon$

Hydraulics radius =R= A/P = $\dfrac{(B + y_0\varepsilon)y_o}{B + 2y_o\varepsilon}$

By manning's formula $Q = \dfrac{1}{n}\left[\dfrac{(B + y_o\varepsilon)^{5/3}y_o^{5/3}}{(B + 2y_o\varepsilon)^{2/3}}\right]S_o^{1/2}$ -- (4.22)

Non-dimensionalising the variables,

$$\frac{Qn\varepsilon^{5/3}}{S_o^{1/2}B^{8/3}} = \phi(\eta_o) = \frac{(1+\eta_o)^{5/3}\eta_o^{5/3}}{(1+2\eta_o)^{2/3}} \quad \text{-----------------} \quad (4.23)$$

For standard triangular case similarly we can drive

$$Q = \frac{1}{n}(\varepsilon y_o^2)(y_o/2)^{2/3}S_o^{1/2} \quad \text{-----------------} \quad (4.24)$$

Example 4.3 A standard lined trapezoidal cannel section is to be designed to convey 100m3/sec of flow. The side slope is to be 1.5H:1V and the manning's coefficient n= 0.016. The longitudinal slope of the bed is 1 in 5000m. If a bed width of 10m is preferred what would be the normal depth?

Solution

Q= 100m3/sec

m= 1.5

n= 0.016

S_o = 0.0002

B= 10m

y_o =?

ε = m+ tan-1(1/m)= 1.5 +tan-1(1/1.5)

= $\underline{\underline{2.088}}$

$$\phi_1 = \frac{Qn\varepsilon^{5/3}}{S_o^{1/2}B^{8/3}} = \frac{100\,X\,0.016\,X\,(2.088)^{5/3}}{(0.0002)^{1/2}(10.0)^{8/3}} = 0.8314$$

$$\phi_1 = \frac{(1+\eta_o)^{5/3}\eta_o^{5/3}}{(1+2\eta_o)^{2/3}} = 0.8314$$

By solving trial and error

$\underline{\eta o = 0.74}$

Then $\eta_0 = \dfrac{y_o\varepsilon}{B}$, which imply that

$$y_0 = \frac{\eta_o B}{\varepsilon} = \frac{0.74X10.0}{2.088}$$

$$y_0 = \underline{\underline{3.554}}$$

The Hydraulic Efficient Channel Section

The *best hydraulic (the most efficient)* cross-section for a given Q, n, and S_0 is the one with a minimum excavation and minimum lining cross-section. $A = A_{min}$ and $P = P_{min.}$ The minimum cross-sectional area and the minimum lining area will reduce construction expenses and therefore that cross-section is ***economically the most efficient*** one. In other case the best hydraulic cross-section for a given A, n, and S_0 is the cross-section that conveys ***maximum discharge***. Thus the cross-section with the ***minimum wetted perimeter*** is the best hydraulic cross-section within the cross-sections with the same area since lining and maintenance expenses will reduce substantially.

$$V = \frac{Q}{A} \rightarrow \frac{Q}{A_{min}} = V_{max}$$

$$V = \frac{1}{n} S_0^{1/2} R^{2/3} = C \times R^{2/3}$$

$$V = V_{max} \rightarrow R = R_{max}$$

$$R = \frac{A}{P}$$

$$R = R_{max} \rightarrow P = P_{min}$$

$$Q = A\frac{1}{n} R^{2/3} S_0^{1/2}$$

$$Q = C' \times R^{2/3}$$

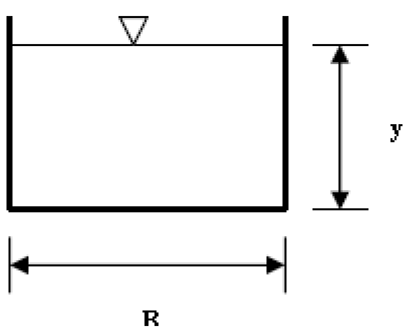
$$C' = const$$

$$Q = Q_{max} \rightarrow R = R_{max}$$

$$R = \frac{A}{P}$$

$$R = R_{max} \rightarrow P = P_{min}$$

Rectangular channel section



$$A = By = \text{Constant}$$

$$B = \frac{A}{y}$$

$$P = B + 2y = \frac{A}{y} + 2y$$

Since P=Pmin for the best hydraulic section, taking the derivative of P with respect to y

$$\frac{dP}{dy} = \frac{\frac{dA}{dy} \times y - A}{y^2} + 2 = 0$$

$$\frac{A}{y^2} = 2 \rightarrow A = 2y^2 = By$$

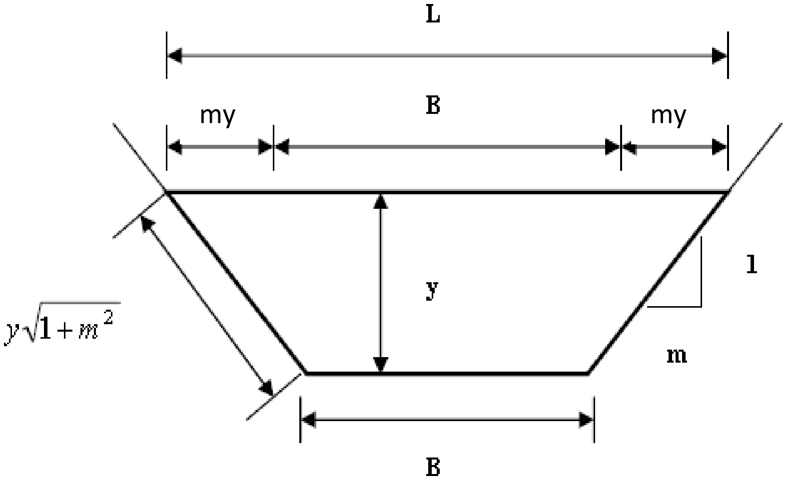
$$B = 2y$$

The best rectangular hydraulic cross-section for a constant area is the one with B = 2y. The hydraulic radius of this cross-section is,

$$R = \frac{A}{P} = \frac{2y^2}{4y} = \frac{y}{2}$$

For all best hydraulic cross-sections, the hydraulic radius should always be R = y/2 regardless of their shapes.

Trapezoidal Channel Section



$$A=\frac{B+B+2my}{y}\times y=(B+my)y$$

$$P=B+2y\sqrt{1+m^2}$$

$$B=\frac{A}{y}-my$$

$$P=\frac{A}{y}-my+2y\sqrt{1+m^2}$$

a). **For a given side slope m, what will be the water depth y for best hydraulic trapezoidal cross-section?**

For a given A , P = P$_{min}$

$$\frac{dP}{dy}=0,\frac{dA}{dy}=0$$

$$\frac{dP}{dy}=\frac{\frac{dA}{dy}y-A}{y^2}-m+2\sqrt{1+m^2}=0$$

$$\frac{A}{y^2}=-m+2\sqrt{1+m^2}$$

$$A=\left(2\sqrt{1+m^2}-m\right)y^2$$

$$P=\left(2\sqrt{1+m^2}-m\right)y-my+2y\sqrt{1+m^2}$$

$$P=2y\sqrt{1+m^2}-my-my+2y\sqrt{1+m^2}$$

$$P=2y\left(2\sqrt{1+m^2}-m\right)$$

The hydraulic radius R, channel bottom width B, and free surface width L may be found as,

$$R=\frac{A}{P}=\frac{\left(2\sqrt{1+m^2}-m\right)y^2}{2y\left(2\sqrt{1+m^2}-m\right)}$$

$$R=\frac{y}{2}$$

$$B=\frac{A}{y}-my=\left(2\sqrt{1+m^2}-m\right)y-my$$

$$B=2y\left(\sqrt{1+m^2}-m\right)$$

$$L=B+2my$$

$$L=2y\left(\sqrt{1+m^2}-m\right)+2my$$

$$L=2y\sqrt{1+m^2}$$

b). **For a given water depth y, what will be the side slope m for best hydraulic trapezoidal cross-section?**

$$P = P_{min}$$

$$\frac{dP}{dm} = 0, \frac{dA}{dm} = 0$$

$$P = \frac{A}{y} - my + 2y\sqrt{1+m^2}$$

$$\frac{dP}{dm} = \frac{\frac{dA}{dm}y - 0}{y^2} - y + 2y\frac{2m}{2\sqrt{1+m^2}} = 0$$

$$y = \frac{2my}{\sqrt{1+m^2}} \rightarrow \frac{2m}{\sqrt{1+m^2}} = 1$$

$$4m^2 = 1 + m^2 \rightarrow 3m^2 = 1$$

$$m = \frac{1}{\sqrt{3}}$$

$$Tan\,\alpha = \frac{1}{m} = \sqrt{3} \rightarrow \alpha = 60^0$$

$$A = \left(2\sqrt{1+m^2} - m\right)y^2$$

$$A = \left(2\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right)y^2$$

$$A = \left(\frac{2\sqrt{4} - 1}{\sqrt{3}}\right)y^2 = \frac{3}{\sqrt{3}}y^2$$

$$A = \sqrt{3}y^2$$

$$P = 2y\left(2\sqrt{1+m^2} - m\right)$$

$$P = 2y\left(2\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right)$$

$$P = 2y\left(\frac{4-1}{\sqrt{3}}\right) = \frac{6y}{\sqrt{3}}$$

$$P = 2y\sqrt{3}$$

$$B = \frac{A}{y} - my = \frac{\sqrt{3}y^2}{y} - \frac{1}{\sqrt{3}}y = \frac{3-1}{\sqrt{3}}y$$

$$B = \frac{2\sqrt{3}}{3}y = \frac{P}{3}$$

$$R = \frac{A}{P} = \frac{\sqrt{3}y^2}{2y\sqrt{3}} = \frac{y}{2}$$

The channel bottom width is equal one third of the wetted perimeter and therefore sides and channel width B are equal to each other at the best trapezoidal hydraulic cross-section. Since α = 60₀, the cross-section is half of the hexagon.

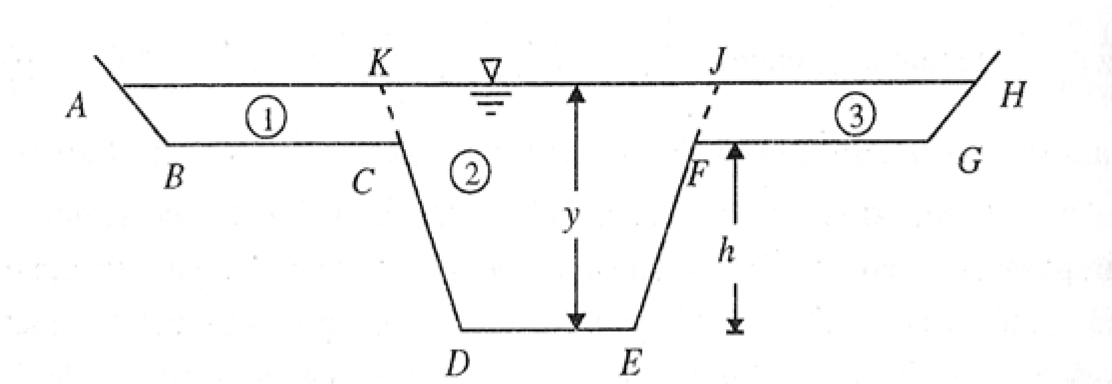
Table 4.6 values of parameters in Efficient (best) hydraulic section

Channel Shape	A	P	B	R	L
Rectangle (half square)	$2y^2$	4y	2y	$\frac{y}{2}$	2y
Trapezoidal (half regular hexagon)	$\sqrt{3}y$	$2\sqrt{3}y$	$\frac{2}{\sqrt{3}}y$	$\frac{y}{2}$	$\frac{4}{\sqrt{3}}y$
Circular (semicircle)	$\frac{\pi}{2}y^2$	πy	-	$\frac{y}{2}$	2y
Triangle (vertex angle $=90^0$)	y^2	$2\sqrt{3}y$	-	$\frac{y}{2\sqrt{2}}$	2y

(A = Area, P = Wetted perimeter, B = Base width,
R = Hydraulic radius, L = Water surface width)

Compound Sections

Some channel sections may be formed as a combination of elementary sections. Typically natural channels, such as rivers, have flood plains which are wide and shallow compared to the main channel. The figure below represents a simplified section of a stream with flood banks. Consider the compound section to be divided into subsections by arbitrary lines. These can be extensions of the deep channel boundaries as in figure. Assuming the longitudinal slope to be same for all subsections, it is easy to see that the subsections will have different mean velocities depending upon the depth and roughness of the boundaries. Generally, overbanks have larger size roughness than the deeper main channel. If the mean velocities Vi in the various subsections are known then the total discharge is ΣViAi.



If the depth of flow is confined to the deep channel only (y < h), calculation of discharge by using Manning's equation is very simple. However, when the flow spills over the flood plain (y > h), the problem of discharge calculation is complicated as the calculation may give a smaller hydraulic radius for the whole stream section and hence the discharge may be underestimated. The following method of discharge estimation can be used. In this method, while calculating the wetted perimeter for the sub-areas, the imaginary divisions (FJ and CK in the Figure) are considered as boundaries for the deeper portion only and neglected completely in the calculation relating to the shallower portion.

1. The discharge is calculated as the sum of the partial discharges in the sub-areas; for e.g. units 1, 2 and 3 in Figuer

$$Q_p = \sum Q_i = \sum V_i A_i$$

2. The discharge is also calculated by considering the whole section as one unit, (ABCDEFGH area in Figure), say Q_w.

3. The larger of the above discharges, Q_p and Q_w, is adopted as the discharge at the depth y.

Design of Irrigation Channels

For a uniform flow in a canal,

$$Q = \frac{1}{n} A R^{2/3} S_0^{0.5}$$

Where A and R are in general, functions of the geometric elements of the canal. If the canal is of trapezoidal cross-section,

$$Q = f(n, y_0, S_0, B, m) \qquad \text{-------------------------------------- (4.25)}$$

Equ. (4.25) has six variables out of which one is a dependent variable and the rest five are independent ones. Similarly, for other channel shapes, the number of variables depends upon the channel geometry. In a channel design problem, the independent variables are known either explicitly or implicitly, or as inequalities, mostly in terms of empirical relationships. The canal-design practice given below is meant only for rigid-boundary channels, i.e. for lined and unlined non-erodible channels.

Canal Section

Normally a trapezoidal section is adopted. Rectangular cross-sections are also used in special situations, such as in rock cuts; steep chutes and in cross-drainage works. The side slope, expressed as *m* horizontal: 1 vertical, depends on the type of canal, (i.e. lined or unlined, nature and type of soil through which the canal is laid). The slopes are designed to withstand seepage forces under critical conditions, such as;

1. A canal running full with banks saturated due to rainfall,
2. The sudden drawdown of canal supply.

Usually the slopes are steeper in cutting than in filling. For lined canals, the slopes roughly correspond to the angle of repose of the natural soil and the values of *m* range from 1.0 to 1.5 and rarely up to 2.0. The slopes recommended for unlined canals in cutting are given in Table (4.7).

Table 4.7: Side slopes for unlined canals in cutting

Type of soil	m
Very light loose sand to average sandy soil	1.5 – 2.0
Sandy loam, black cotton soil	1.0 – 1.5
Sandy to gravel soil	1.0 – 2.0
Murom, hard soil	0.75 – 1.5
Rock	0.25 – 0.5

Longitudinal Slope

The longitudinal slope is fixed on the basis of topography to command as much area as possible with the limiting velocities acting as constraints. Usually the slopes are of the order of 0.0001. For lined canals a velocity of about 2 m/sec is usually recommended.

Roughness coefficient n

Procedures for selecting *n* are discussed and values of n can be taken from Table (4.4).

Permissible Velocities

Since the cost for a given length of canal depends upon its size, if the available slope permits, it is economical to use highest safe velocities. High velocities may cause scour and erosion of the boundaries. As such, in unlined channels the maximum permissible velocities refer to the velocities that can be safely allowed in the channel without causing scour or erosion of the channel material. In lined canals, where the material of lining can withstand very high velocities, the maximum permissible velocity is determined by the stability and durability of the lining and also on the erosive action of any abrasive material that may be carried in the stream. The permissible maximum velocities normally adopted for a few soil types and lining materials are given in Table (4.8).

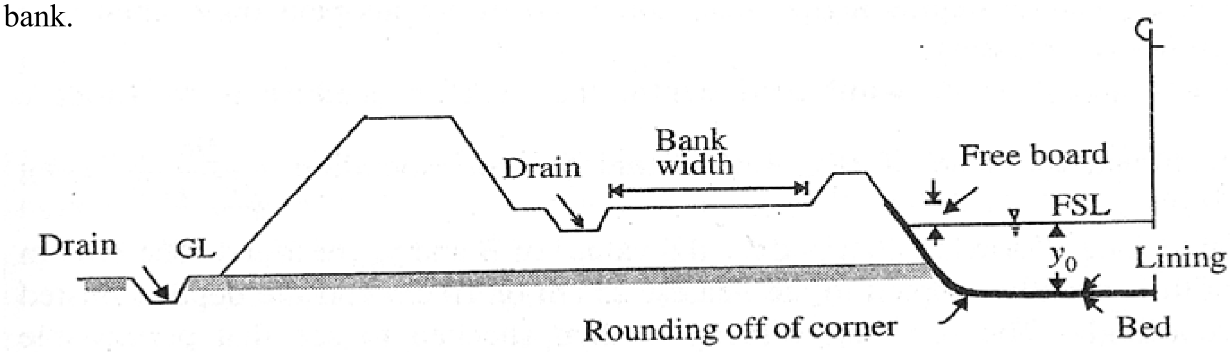
Table 4.8 Permissible Maximum velocities

Nature of boundary	Permissible maximum velocity (m/sec)
Sandy soil	0.30 – 0.60
Black cotton soil	0.60 – 0.90
Hard soil	0.90 – 1.10
Firm clay and loam	0.90 – 1.15
Gravel	1.20
Disintegrated rock	1.50
Hard rock	4.00
Brick masonry with cement pointing	2.50
Brick masonry with cement plaster	4.00
Concrete	6.00
Steel lining	10.00

In addition to the maximum velocities, a minimum velocity in the channel is also an important constraint in the canal design. Too low velocity would cause deposition of suspended material, like silt, which cannot only impair the carrying capacity but also increase the maintenance costs. Also, in unlined canals, too low a velocity may encourage weed growth. The minimum velocity in irrigation channels is of the order of 0.30 m/sec.

Free Board

Free board for lined canals is the vertical distance between the full supply levels to the top of lining (Fig. 4.17). For unlined canals, it is the vertical distance from the full supply level to the top of the bank.



This distance should be sufficient to prevent overtopping of the canal lining or banks due to waves. The amount of free board provided depends on the canal size, location, velocity and depth of flow.

Width to Depth Ratio

The relationship between width and depth varies widely depending upon the design practice. If the hydraulically most-efficient channel cross-section is adopted,

$$m = \frac{1}{\sqrt{3}} \rightarrow B = \frac{2y_0}{\sqrt{3}} = 1.155y_0 \rightarrow \frac{B}{y_0} = 1.155$$

If any other value of m is use, the corresponding value of B/y0 for the efficient section would be

$$\boxed{\frac{B}{y_0} = 2\left(2\sqrt{1+m^2} - m\right)}$$

In large channels it is necessary to limit the depth to avoid dangers of bank failure. Usually depths higher than about 4.0 m are applied only when it is absolutely necessary. For selection of width and depth, the usual procedure is to adopt a recommended value.

Example 4.4.